Objectives:

- Find local maxima and minima of a function graphically.
- Find the absolute maximum and the absolute minimum of a function graphically.
- Use the first derivative test to find local extrema analytically.

Definitions:

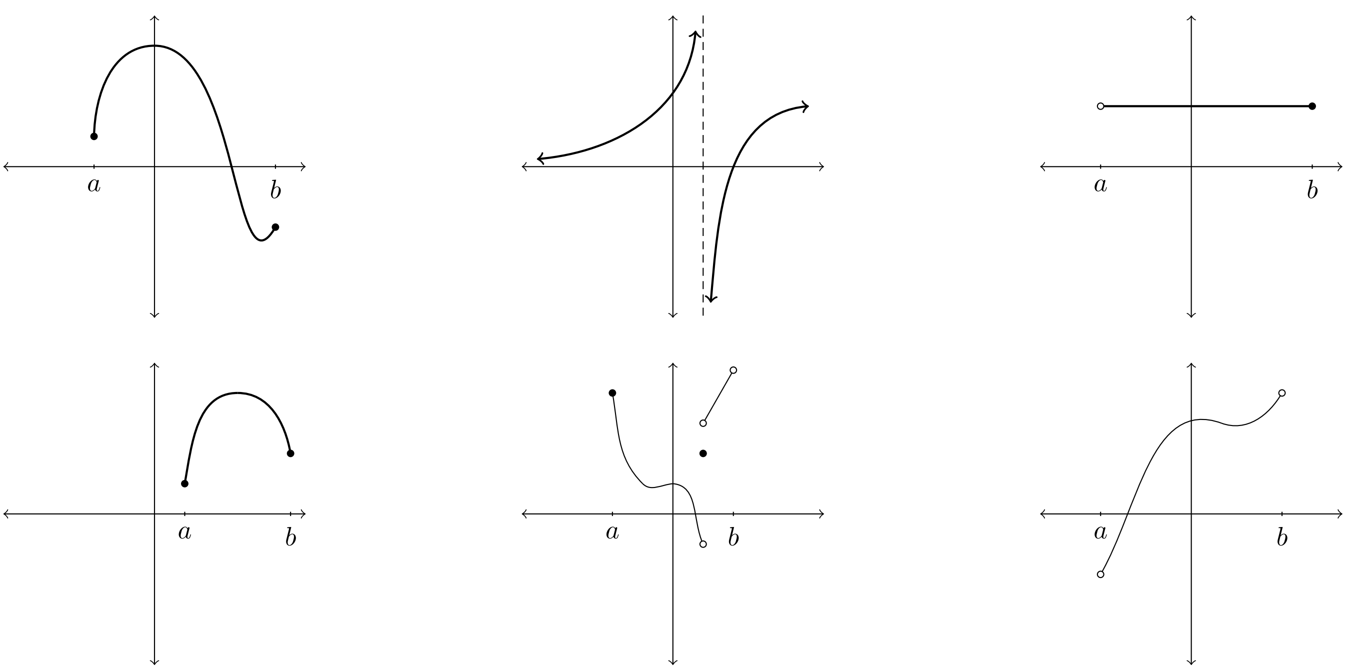
- CRITICAL NUMBER: A number c in the domain of a function f is a **critical number** of f if

- ABSOLUTE MAXIMUM VALUE: For a number c in the domain of a function f, the number $f(c)$ is the **absolute maximum value** of $f(x)$ if

- ABSOLUTE MINIMUM VALUE: For a number c in the domain of a function f, the number $f(c)$ is the **absolute minimum value** of $f(x)$ if

We refer to absolute maximum and minimum values as ____________________________________ .

Functions don't always have absolute extrema. Which of these functions have absolute extrema?



You will work more with absolute extrema in your project tomorrow!

Definitions:

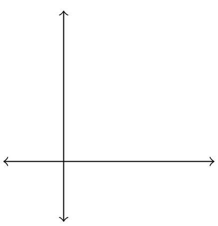
- LOCAL MAXIMUM VALUE: The number $f(c)$ is a **local maximum value** of $f(x)$ if

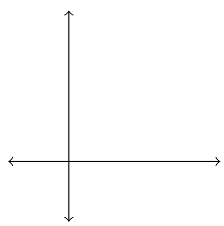
- LOCAL MINIMUM VALUE: The number $f(c)$ is a **local minimum value** of $f(x)$ if

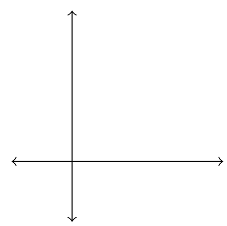
We refer to local maximums and minimums as ________________________________ .
If we are on a hunt for local extrema, where should we look?

Fermat's Theorem:

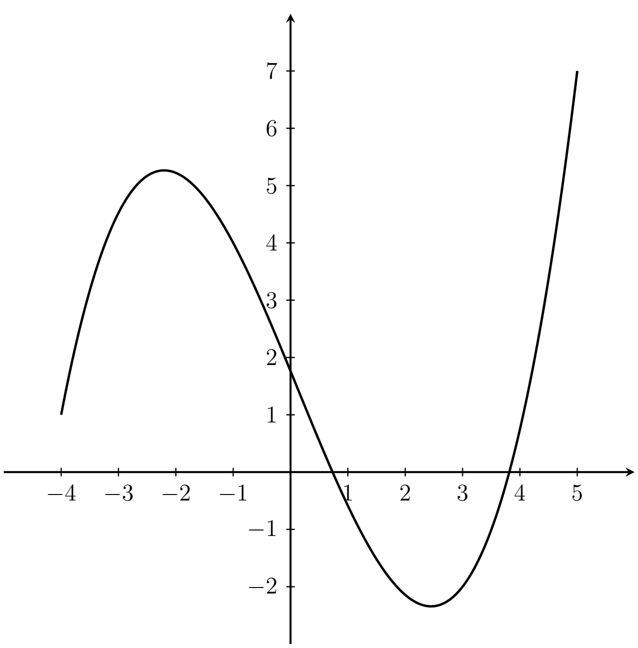
CAUTION: the converse is not true! If f has a critical number c, f doesn't necessarily have a local extreme value there:







Graphical example:



So, if c is a critical number for f, how do we tell if f has a local max, local min, or neither?

Analytically, there are two options (described in Section 4.3). Here is the first one:

First Derivative Test: If c is a critical number for a continuous function f,

(a) If $f'(x)$ changes from positive to negative at c, then ____________________________ .

(b) If $f'(x)$ changes from negative to positive at c, then ____________________________ .

(c) If $f'(x)$ does not switch signs, then ____ .

Example: Find and classify all critical numbers of

$$f(x) = 2x^3 + 3x^2 - 12x.$$

Steps:

1. Find $f'(x)$.

2. Find where $f'(x) = 0$ or where $f'(x)$ DNE.

3. Make a sign chart for $f'(x)$.

4. Use the first derivative test to classify each critical number.